Internet Advertising and Generalized Second Price Auctions

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November / Econ415
• What is sold and why. Who buys.
• Background: Progression of Search Page Advertising
• A Generalized Second Price Auction and VCG Mechanisms
• Equilibrium in GSP.
• A Generalized English Auction.
• Try to search ‘Hawaii vacations’.
• And Again..
Development of Position Advertising

- Initial internet ads took the form of impressions on pages and then banner ads. They allowed users to click on the ads to go to advertiser-selected sites. The original banner ad (ATT?)

- With the exception of identifying readers of the web pages, these ads were ‘untargeted’. But became popular as major advertisers also developed their own websites as destinations.

- Firms emerged (like Doubleclick) that allowed advertisers to track who clicked on their ads and what their purchase history was.

- Instead of cost per impression, cost per click became the norm.
Development of Position Advertising

- Pop-up and pop under ads replaced banner ads and were somewhat more targeted.
- In 2002, Google developed Adwords as a means of coordinating advertisers on their search pages. It was originally a flat CPM price system that did not succeed. It adopted an auction based system in 2002.
Search Engines: A new form of Web Content

• The rapid growth of the internet in the 1990s led to a need to find a way to navigate the web and find content.
• Yahoo began with a ‘directory’ which was a hand constructed index of places on the web. Excite, Infospace, Jeeves, Ask.com, AltaVista
• This was far too primitive for the task at hand and Google’s primary innovation was developing a web crawler that would automate the search for web content (along with a clever ranking algorithm based on links).
• This developed into the search engine, a site whose function is to find targeted content on the web based on keywords.
• Bing, Google and Yahoo are the primary search engine sites.
• Google earned over $52 B in 2015 from web search ads. ($38B in second quarter 2019).
Development of Position Advertising

- Search pages are just another form of web content.
- However, advertising is a more targeted way to reach consumers. Advertisers expect that consumers searching on particular ‘keywords’ will be more receptive to certain messages.
- Advertising departments must come up with keywords they wish to bid on.
- To value the bid, they need to determine propensity of viewers to buy after they click on a link.
- In order to evaluate whether to display advertisers, search engines like Google have to determine the likelihood the advertiser will generate a click.
- Google applies a ‘quality score’ to advertisers for each keyword. It maintains a pool of advertisers in real time.
- Note that a sponsored search ad is a highly ‘perishable’ good. If an ad is not displayed, the eyeball is lost.
• Original banner ads were simply posted or negotiated prices.
• As competition grew for placement, .
• Online search engines began to develop auctions for placement.
• Original auctions were pay your bid auctions, with the exception that payment was only due upon a click occurring. (Requires a click monitoring technology.)
• Since auctions were conducted in real time, there was a great incentive for bidders to continually revise their bids in response to other bidders.
• Many resources were spent on trying to be the first bidder to adjust.
A Generalized Second Price Auction (GSP) was a move away from pay your bid.

Suppose there are $K$ slots to be auctioned for a given keyword.

Slots are ordered according to likelihood of click.

Each advertiser submits a bid and bids are ordered from highest to lowest.

Highest bid wins the top slot and pays the bid of the second highest, in general, $k$th bid wins the $k$th slot and pays the bid of the $k + 1$st bid.
- Note that we could have used a $K + 1$st price auction.
- A version of the second price auction.
- Top $K$ bidders win and all pay the $K + 1$st price.
- But who gets the better slots?
- If highest bidder, then now bid your value is not a good strategy.
One-sided Matching?

- How might a matching algorithm work?
- How does each advertiser rank slots?
- What would the outcome be for each serial dictatorship?
- What about for each TTC given any initial assignment?
Modelling Advertiser Preferences

- Advertisers $k \in K$, differ in terms of how valuable, $s_k$, each views a given click.
- Slots $i \in N$ are differentiated by the number of clicks $\alpha_j$ an exposure in a given slot is expected to generate.
- Order slots so that $\alpha_j > \alpha_{j+1}$.
- Thus an advertiser $k$ values a slot $i$ at $s_k \alpha_i$.
- If we order advertisers so that $b_k > b_{k+1}$, the net payoff to advertiser $k$ is

$$ (s_k - b_{k+1}) \alpha_i. $$
In a Vickrey Auction, advertisers bid their true values, $b_k = s_k$ and slots are allocated so that $\alpha_i$ goes to the $i$th highest bidder.

Assume $K > N$ so there are more advertisers than slots.

Bidder, $l$ displaced bidder $l + 1$ and so the total surplus of everyone else with bidder $l$ present is

$$\sum_{k=1}^{l-1} \alpha_k s_k + \sum_{k=l+1}^{N} \alpha_k s_k.$$ 

The total surplus of everyone else with bidder $l$ absent is

$$\sum_{k=1}^{k} \alpha_k s_k + \sum_{k=l+1}^{N+1} \alpha_{k-1} s_k.$$
• Therefore bidder $i$’s payment is ($\alpha_{N+1} \equiv 0$)

$$
\sum_{k=l+1}^{N+1} (\alpha_{k-1} - \alpha_k)s_k = (\alpha_l - \alpha_{l+1})s_{l+1} + \sum_{k=l+2}^{N+1} (\alpha_{k-1} - \alpha_k)s_k.
$$

• Notice that in a Vickrey Auction, the difference between the $i$th bidder payment and the $i + 1$st payment is

$$
(\alpha_i - \alpha_{i+1})b_{i+1} = (\alpha_i b_{i+1} - \alpha_{i+1} b_{i+1})
$$

• If bidders bid their true values in a GSP auction, the difference is

$$
(\alpha_i b_{i+1} - \alpha_{i+1} b_{i+2})
$$

• Since bidder $N$ pays the same in each auction this implies that if bidders bid truthfully in each auction, payment would be higher in the GSP.
But would bidders bid truthfully in a GSP?

Suppose \( K = \{1, 2, 3\} \) and \( N = 2 \).

\( s_1 = 10, s_2 = 4, s_3 = 2. \ \alpha_1 = 200, \alpha_2 = 199. \)

If all bidders bid their true values, bidder 1’s payoff is
\[ 200(10 - 4) = 1200. \]

But if bidder 1 bids 3 and accepts the second slot, its payoff is
\[ 199(10 - 2) = 1592 \] so bidder 1 would be better to underbid.
• Suppose that all bidders know each other’s type.
• In general, there are many equilibria of this game. EOS highlight one interesting equilibrium.
• Order bidders by value from $k = 1, \ldots, k + K$. Let $V^k$ be the payment of bidder $k$ in the Vickrey mechanism.
• $b_k = V^{k-1}/\alpha_{k-1}$. $b_1 = s_1$. Thus, the actual payment is the Vickrey payment.
• These bids satisfy a sort of local envy freeness each bidder prefers its slot and payment to the one above it:

\[ \alpha_i s_i - p_i = \alpha_{i-1} s_{i-1} - p_{i-1}. \]
A defect of the above argument is that it relies on bidders knowing each other’s valuations to play it. EOS consider an alternative dynamic auction which they call a Generalized English Auction.

In this auction, bidders do not know each others values and it operates like a clock auction.

Clock starts at a low per click bid price and rises continuously. Bidders hold a button and drop out irrevocably when they lift the button.

When there are \(N\) bidders remaining, the next bidder to drop out gets the \(N\)th place at the per click price of the \(N + 1\) bidder drop out point.

More generally, when there are \(k \leq N\) bidders left, the next bidder to drop out pays the \(k + 1\) price.

Bidders see the drop out prices as the clock rises.
Equilibrium in GEAs?

An equilibrium strategy of a bidder with \( i \) bidders left, given the most recent drop out price \( b_{i+1} \) is to select a drop-out point.

Note, for any bidder remaining in the auction, a new drop-out price has to be selected for each different \( i \) and \( b_i \). Why?

In the equilibrium EOS identify, bidders choose a drop-out price so they are just indifferent between getting the \( i - 1 \)st slot at the drop-out price and the \( i \)th slot at the price, \( b_{i+1} \):

\[
\alpha_{i-1}(s_k - p) = \alpha_i(s_k - b_{i+1})
\]

This means

\[
p = s_k - \frac{\alpha_i}{\alpha_{i-1}}(s_k - b_{i+1}).
\]
These bids are increasing in $s_k$ for every remaining set $i$ and every $b_{i+1}$ so the highest valuation advertisers get the most valuable slots.

The equilibrium payment of each bidder corresponds to the payment it would make in a Vickrey auction.

The strategies are not dominant, however, they are ‘ex post’:

Given what other bidders have done, no bidder would want to alter its strategy even if it learned the valuations of the other bidders.

Compare to pay your bid auctions for example.